

The boundary of the first order chiral phase transition in the $m_\pi - m_K$ -plane with a linear sigma model

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BNL, May 13, 2006

- Motivation for the study of the phase-boundary in the $m_\pi - m_K$ -plane
- Tree-level and complete one-loop level parametrization of the L σ M
- Boundary of the first order transition region and the location of the TCP
- Conclusions

Why and how to study the boundary with effective models ?

current lattice investigations indicate crossover transition at the physical point

Z. Fodor, S. D. Katz, JHEP 0404:050

Lattice studies of thermodynamics with 3 degenerate quarks $m_u = m_d = m_s$

$$m_\pi^c \approx 290 \text{ MeV}$$

Karsch *et al.* PLB520:41

$$m_\pi^c \approx 270 \text{ MeV}$$

Christ & Liao NPB(PS)119:514

$$m_\pi^c = 67(18) \text{ MeV}$$

Karsch *et al.* NPB(PS)129-130:614

Effective model: $SU(3) \times SU(3)$ $L_\sigma M$

Goldstone masses are tuned with explicit symmetry breaking terms.

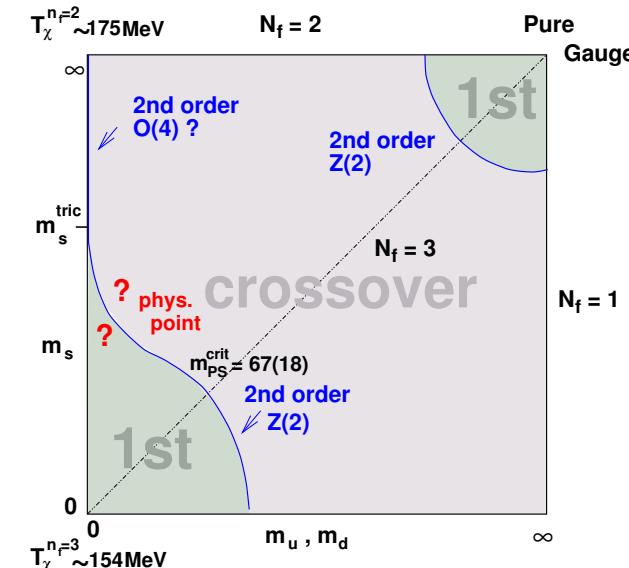
Other parameters are kept at the values determined in the physical point.

$$m_\pi^c = 51 \text{ MeV}$$

Meyer-Ortmanns & Schaefer PRD53:6586

$$m_\pi^c = 47 \text{ MeV}$$

C. Schmidt (2003) Ph.D thesis [see J.T. Lenaghan PRD 63:048901]



Effective model studies might complement lattice investigations if coupling parameters can be determined accurately.

Chiral Perturbation Theory

Describes the dynamics of Goldstone particles

Gasser & Leutwyler

$$U(\Phi) = e^{i(\sqrt{2}\Phi + \eta_0\lambda^0)/f} \quad \Phi(x) = \begin{pmatrix} \pi^0/\sqrt{2} + \eta_8/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6} \end{pmatrix}$$

$$\det U = \exp(i\sqrt{6}\eta_0/f)$$

Lagrangian organized in a systematic expansion in p, m_q

$$\mathcal{L}_{\text{eff}}^{SU(3)}(U) = \sum_n \mathcal{L}_{2n} = \frac{f^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f^2 B_0}{2} \text{Tr}(M U^\dagger + U M^\dagger) + \mathcal{O}(p^4),$$

$$3f^2 B_0 = -\langle \bar{q}q \rangle$$

Important result: m_π and m_K dependence of decay constants:

$$f_\pi = f \left\{ 1 - [2\mu_\pi + \mu_K - 4m_\pi^2(L_4 + L_5) - 8m_K^2 L_4]/f^2 \right\}$$

$$f_K = f \left\{ 1 - [3(\mu_\pi + \mu_\eta + 2\mu_K)/4 - 4m_\pi^2 L_4 - 4m_K^2(L_5 + 2L_4)]/f^2 \right\}.$$

and of mass eigenvalues $m_\eta^2, m_{\eta'}^2$ Herrera-Siklódy et al. ($\frac{1}{N_c}$); Beisert & Borasoy (U(3))

μ_i : Chiral logarithms determined by the masses and the scale $M_0 = 4\pi f$

L_i : determined by masses and decay constants in the physical point.

$SU_L(3) \times SU_R(3)$ symmetric linear sigma model

$$\begin{aligned}\mathcal{L}(\Phi) = & \text{Tr}(\partial\Phi^\dagger\partial\Phi + \mu^2\Phi^\dagger\Phi) - f_1(\text{Tr}(\Phi^\dagger\Phi))^2 - f_2\text{Tr}(\Phi^\dagger\Phi)^2 \\ & - g[\det(\Phi) + \det(\Phi^\dagger)] + h_0\sigma_0 + h_8\sigma_8\end{aligned}$$

$$\Phi = \lambda_a(\sigma_a + i\pi_a) \quad 3 \times 3 \text{ complex matrix} \quad \begin{aligned}\sigma_a &\rightarrow \bar{q}\lambda_a q & (J^P = 0^+) \\ \pi_a &\rightarrow \bar{q}\lambda_a \gamma_5 q & (J^P = 0^-)\end{aligned}$$

$\lambda_a, a = 0 \dots 8$: Gell–Mann matrixes

determinant breaks $U_A(1)$ symmetry

explicit symmetry breaking: external fields $h_0, h_8 \neq 0 \iff m_u = m_d \neq 0, m_s \neq 0$

broken symmetry phase: two condensates $(\langle\sigma_0\rangle, \langle\sigma_8\rangle) \longleftrightarrow (x, y)$

x: non-strange, y: strange

at tree-level: $h_x = m_\pi^2 x, \quad h_y = \frac{1}{\sqrt{2}}(m_K^2 - m_\pi^2)x + m_K^2 y$

technical difficulty: mixing in the x, y sector

parameters determined from the $T = 0$ mass spectra

Tree-level parametrization

Basic idea: Use of ChPT in moving away from the physical point

input: **output:** **prediction:** parameters determined through a set of coupled linear equations

$$\left. \begin{array}{l} f_\pi \\ f_K \\ m_\pi \\ m_K \\ M_\eta^2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x \\ y \\ g \\ f_2 \\ M^2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} m_\eta \\ m_{\eta'} \\ \theta_\eta \\ m_{a_0} \\ m_\kappa \end{array} \right.$$

disentangling f_1 and μ^2 in $M^2 = -\mu^2 + 4f_1(x^2 + y^2)$ is possible only using the scalar sector!

$$\left. \begin{array}{l} A1 \& M^2 \\ A2 \& M^2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \mu^2 \\ f_1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} m_\sigma \\ m_{f_0} \\ \theta_\sigma \end{array} \right.$$

$$\left. \begin{array}{l} E_x = 0 \\ E_y = 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} h_x \\ h_y \end{array} \right.$$

$$M_\eta^2 = m_\eta^2 + m_{\eta'}^2$$

⇒ assumption is needed

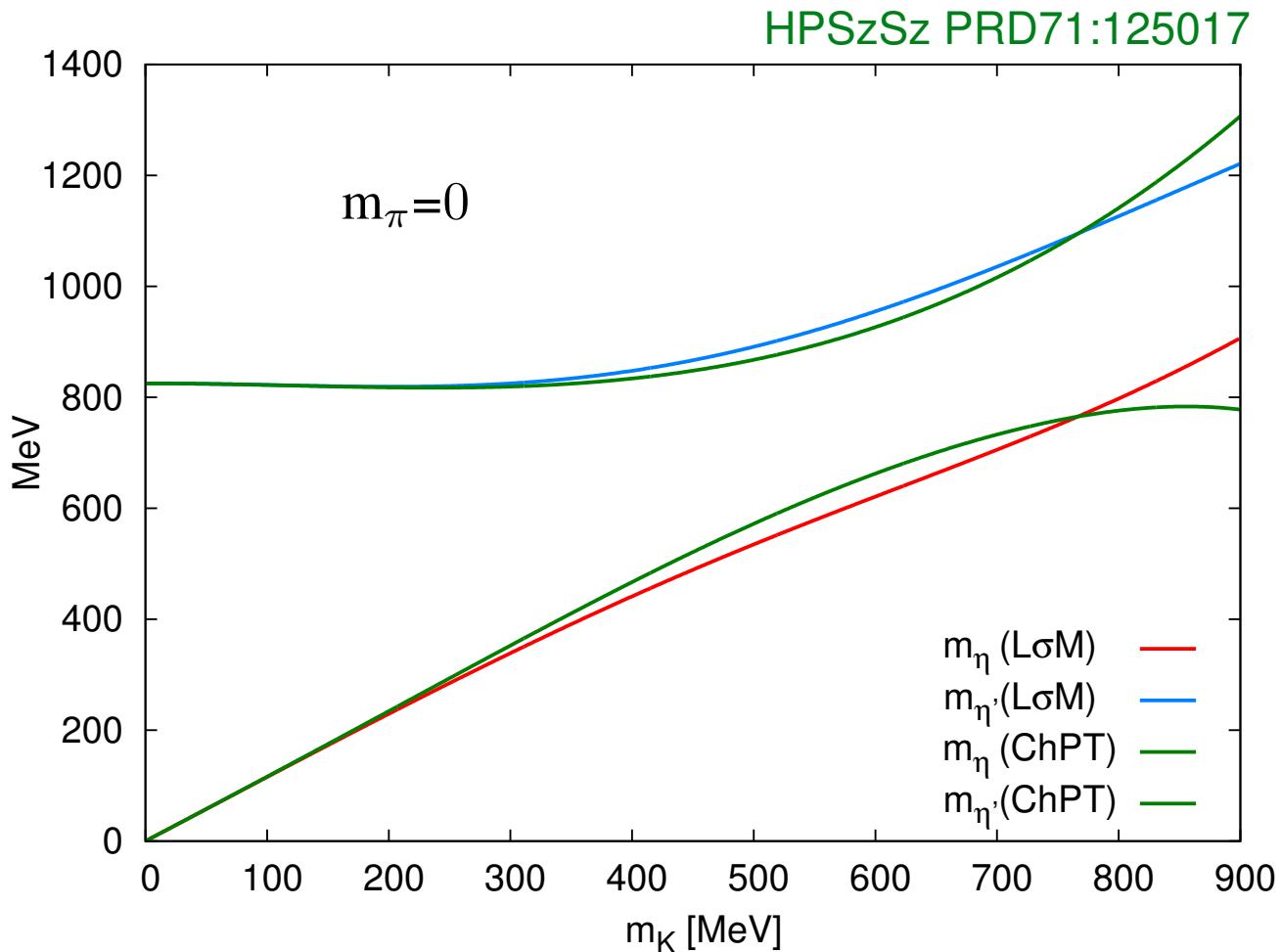
Alternatives:

A1 Requiring no $x - y$ mixing in the scalar meson sector

A2 Requiring Gell-Mann–Okubo relation in the scalar sector to be satisfied in the full $(m_\pi - m_K)$ -plane with prescribed accuracy

cf. N.A. Törnqvist, EPJ C11:359
 Lenaghan et al., PRD62,085008

Consistency check: m_π and m_K dependence of m_η^2 , and $m_{\eta'}^2$, obtained in 3-flavor linear σ model and in $U(3)$ ChPT, respectively



Remarkable agreement up to $m_K \approx 800$ MeV

Parametrization at one-loop level

resummation using optimized perturbation theory Chiku & Hatsuda, PRD58:076001

shift: $-\mu^2 \rightarrow M^2 \Rightarrow L_{mass} = -\frac{1}{2}M^2 \text{Tr}\Phi^\dagger\Phi + \frac{1}{2} \underbrace{(\mu^2 + M^2) \text{Tr}\Phi^\dagger\Phi}_{\Delta m^2: \text{one-loop counterterm}}$

principle of minimal sensitivity $M_\pi^2 = iG^{-1}(p^2 = M_\pi^2) \Big|_{\text{1-loop}} \stackrel{!}{=} m_\pi^2 \Big|_{\text{tree}}$

\implies self-consistent gap equation for the pion mass

Set of coupled nonlinear equations:

(1) gap-equation $m_\pi^2 = -\mu^2 + (4f_1 + 2f_2)x^2 + 4f_1y^2 + 2gy + \text{Re}\Sigma_\pi(p^2 = m_\pi^2)$

(2) pole-mass $M_K^2 = -\mu^2 + 2(2f_1 + f_2)(x^2 + y^2) + 2f_2y^2 - \sqrt{2}x(2f_2y - g) + \text{Re}\Sigma_K(p^2 = M_K^2)$

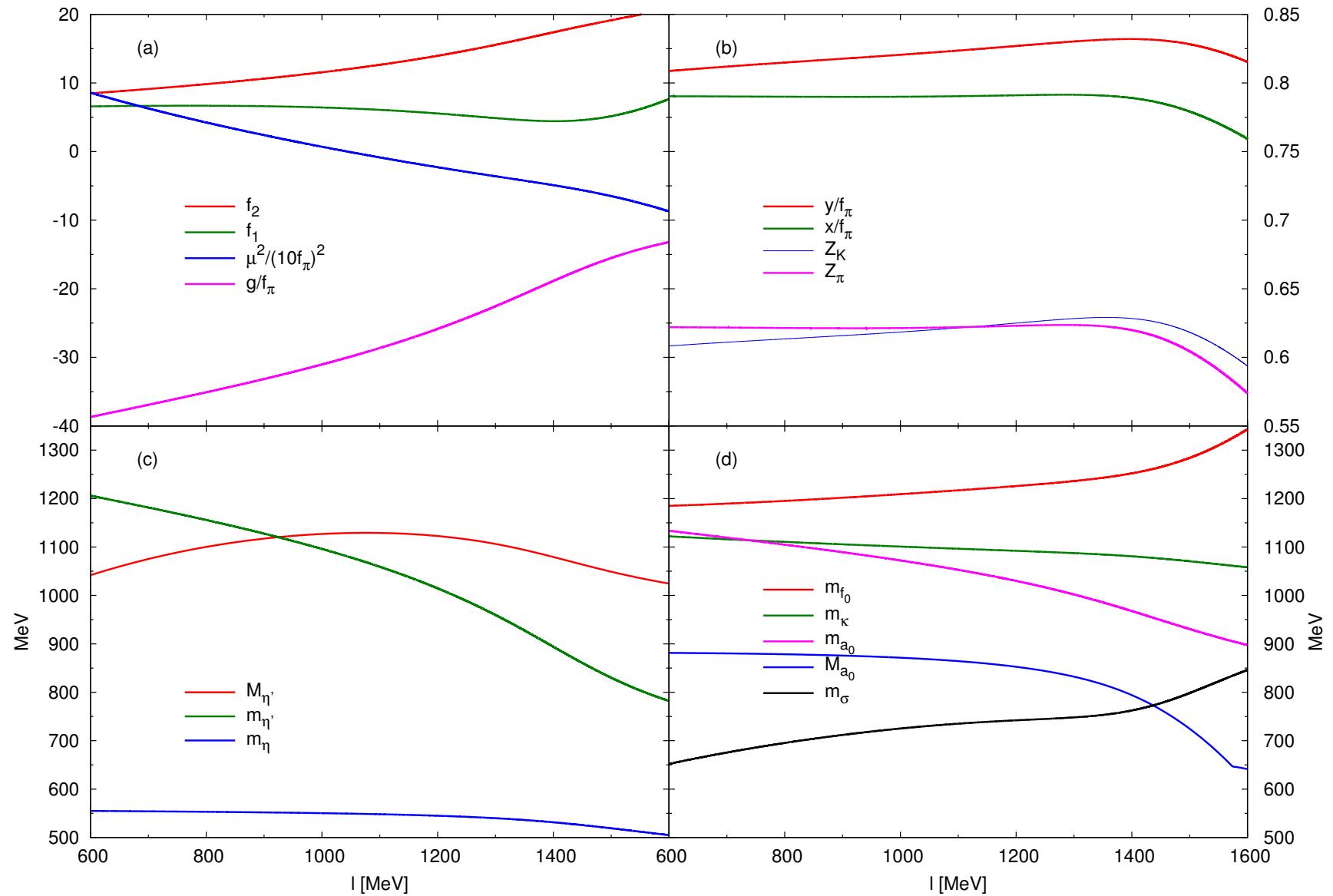
(3) pole-mass M_η from $\text{Det} \begin{pmatrix} p^2 - m_{\eta xx}^2 - \Sigma_{\eta xx}(p^2) & -m_{\eta xy}^2 - \Sigma_{\eta xy}(p^2) \\ -m_{\eta xy}^2 - \Sigma_{\eta xy}(p^2) & p^2 - m_{\eta yy}^2 - \Sigma_{\eta yy}(p^2) \end{pmatrix} \Big|_{p^2 = M_\eta^2, M_{\eta'}^2} = 0$

(4) $M_K^2 \stackrel{!}{=} m_K^2 = m_\pi^2 - 2gy + 4f_2y^2 - \sqrt{2}x(2f_2y - g)$

(5-6) PCAC $f_\pi = Z_\pi^{-\frac{1}{2}} \frac{-iD_\pi^{-1}(p=0)}{M_\pi^2} x, \quad f_K = Z_K^{-\frac{1}{2}} \frac{-iD_K^{-1}(p=0)}{M_K^2} \frac{x + \sqrt{2}y}{2}$.

m_η, f_π, f_K determined using $SU(3)$ ChPT in the large N_c -limit

Renormalization scale dependence of the parameters



moderate dependence of Z_π , Z_K and mass spectra for $l \in (1000, 1400)$ MeV

One-loop resummed thermodynamics

tree-level parametrization enforces the use of **quasi-particle approximation**
 \implies no vacuum fluctuation

one-loop parametrization **includes** renormalized vacuum fluctuations
 dependence on the renormalization scale has to be investigated

Order of the phase transition determined solving:

(1) self-consistent gap equation for the pion mass

$$m_\pi^2 = -\mu^2 + (4f_1 + 2f_2)\mathbf{x}^2 + 4f_1\mathbf{y}^2 + 2g\mathbf{y} + \Sigma_\pi(p^2 = m_\pi^2, m_i(m_\pi), l)$$

$$\sum_\pi(p, m_i(m_\pi), l) = \sum_{i=\pi, K, \eta, \eta'} \pi \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ i \end{array} \pi + \sum_{i=a_0, \kappa, \sigma, f_0} \pi \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ i \end{array} \pi + \sum_{i=a_0, \sigma, f_0} \pi \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \pi \\ i \end{array} \pi + \sum_{i=\eta, \eta'} \pi \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ a_0 \\ i \end{array} \pi + \pi \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ K \\ \kappa \end{array} \pi + \frac{\pi \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \emptyset \end{array} \pi}{\Delta m^2}$$

(2) equations of state:

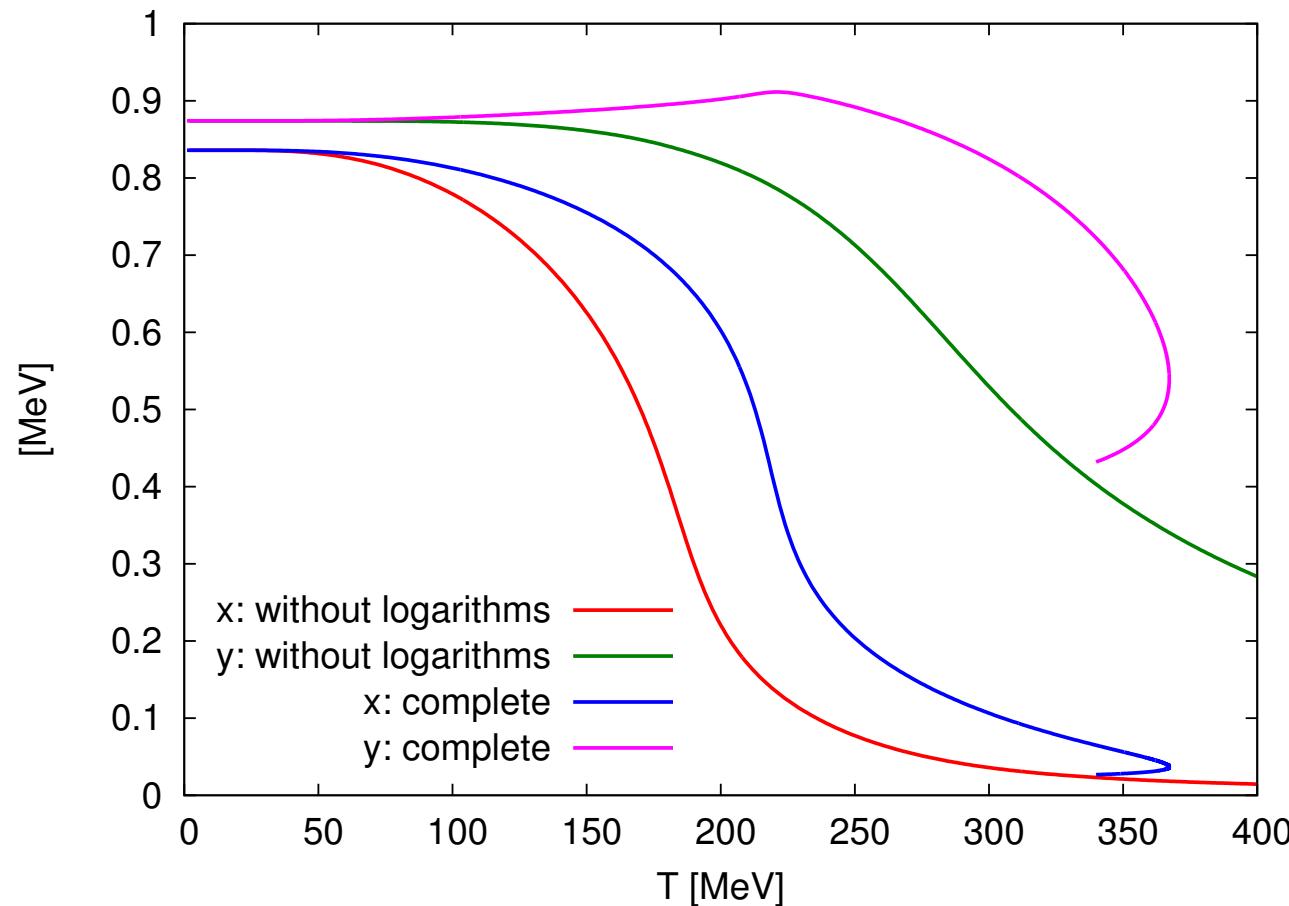
$$\begin{aligned} h_x &= -\mu^2 + 2g\mathbf{xy} + 4f_1\mathbf{xy}^2 + (4f_1 + 2f_2)\mathbf{x}^3 + \sum_i t_i^x(\mathbf{x}, \mathbf{y}) \mathbf{I}_{\text{tp}}[m_i(m_\pi, \mathbf{x}, \mathbf{y}), \mathbf{T}] \\ h_y &= -\mu^2 + 2g\mathbf{x}^2 + 4f_1\mathbf{x}^2\mathbf{y} + (4f_1 + 4f_2)\mathbf{y}^3 + \sum_i t_i^y(\mathbf{x}, \mathbf{y}) \mathbf{I}_{\text{tp}}[m_i(m_\pi, \mathbf{x}, \mathbf{y}), \mathbf{T}] \end{aligned}$$

OPT preserves relations among tree-level masses \Rightarrow Goldstone theorem for pions is satisfied but for kaon is violated because $m_K \neq M_K$.

Influence of the logarithmic terms

one-loop parameters taken with modified μ^2 incorporating $T = 0$ logarithmic terms

difference in the thermodynamics with one-loop parametrization and with modified one-loop (tree-level) parametrization due to the T -dependence of logarithmic terms

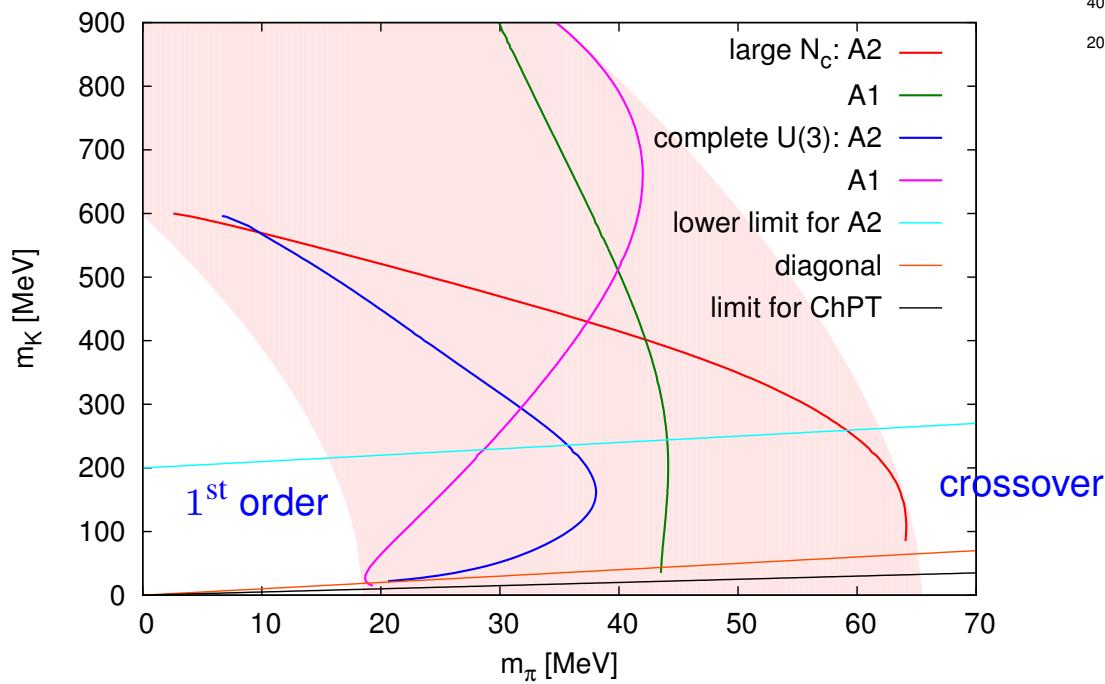


solution reliable only up to $T \simeq 350$ MeV

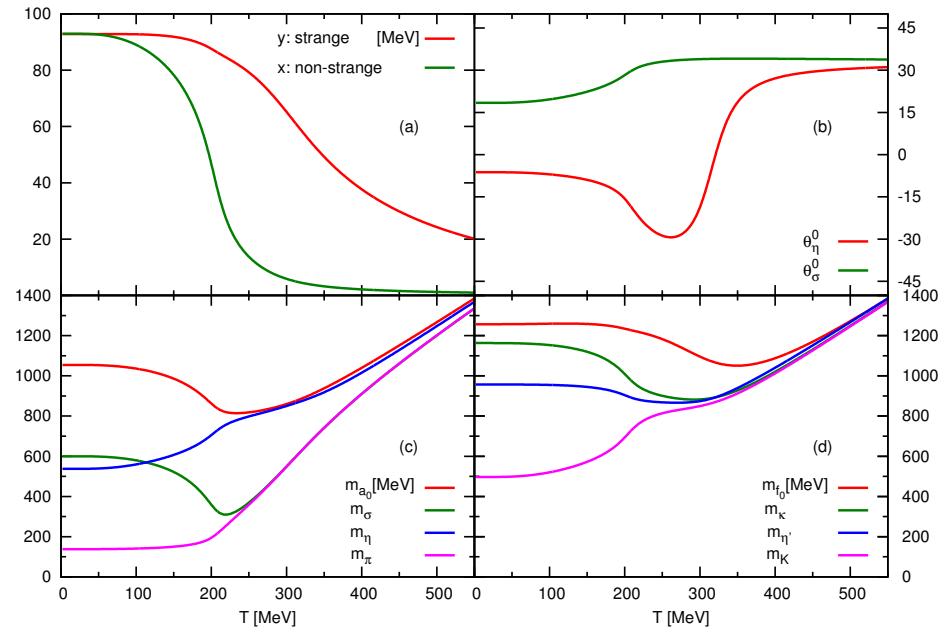
transition region not affected

Phase boundary with tree-loop parametrization

Boundary lines



Physical point



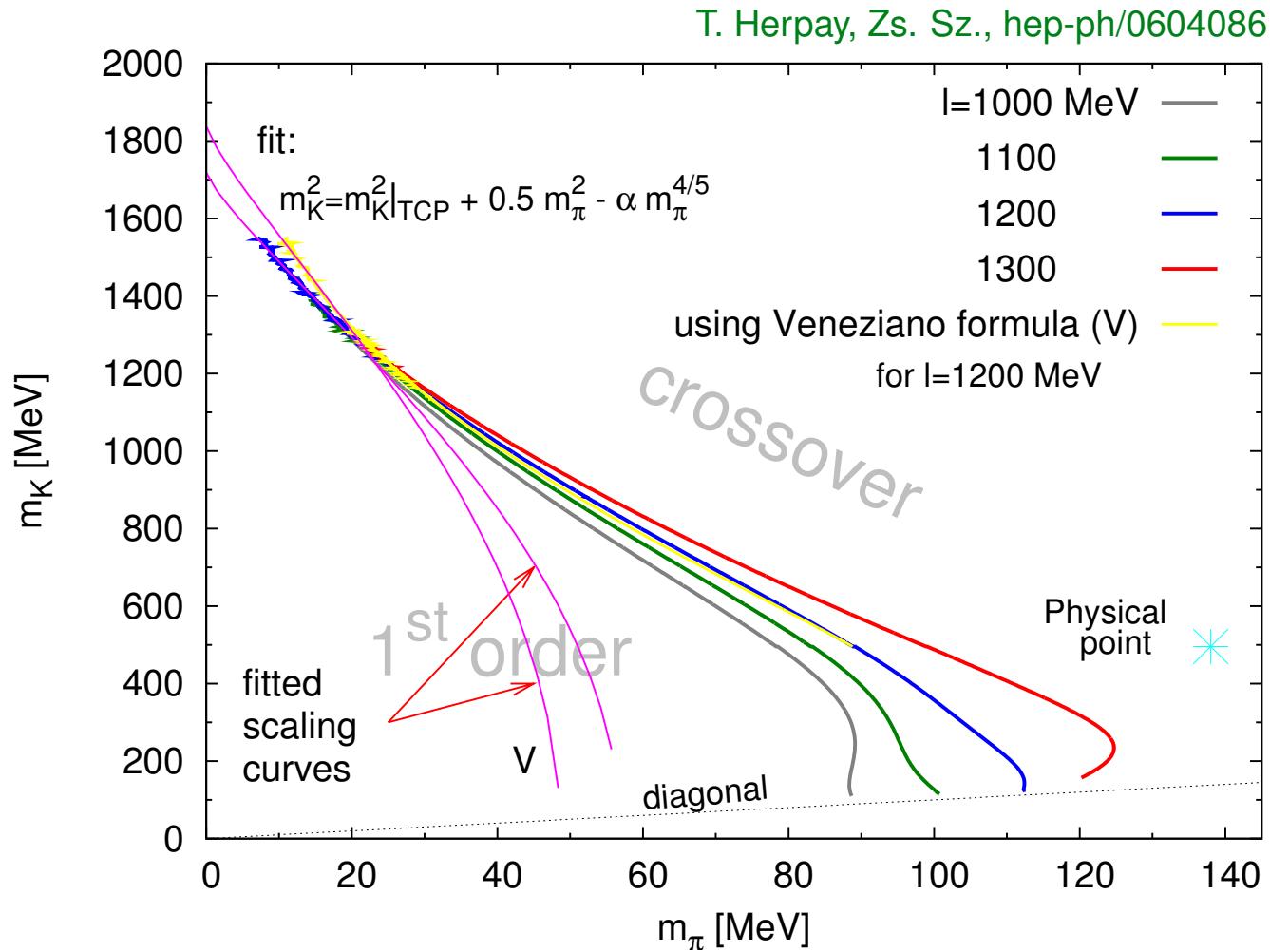
- a) condensates
- b) mixing angles
- c) d) tree-level masses

uncertainty due to strong dependence on the scalar sector
estimate for $m_\pi = m_K$: $20 \text{ MeV} < m_\pi^c < 65 \text{ MeV}$

Phase boundary with one-loop parametrization

estimate for $m_\pi = m_K$: $90 \text{ MeV} < m_\pi^c < 130 \text{ MeV}$

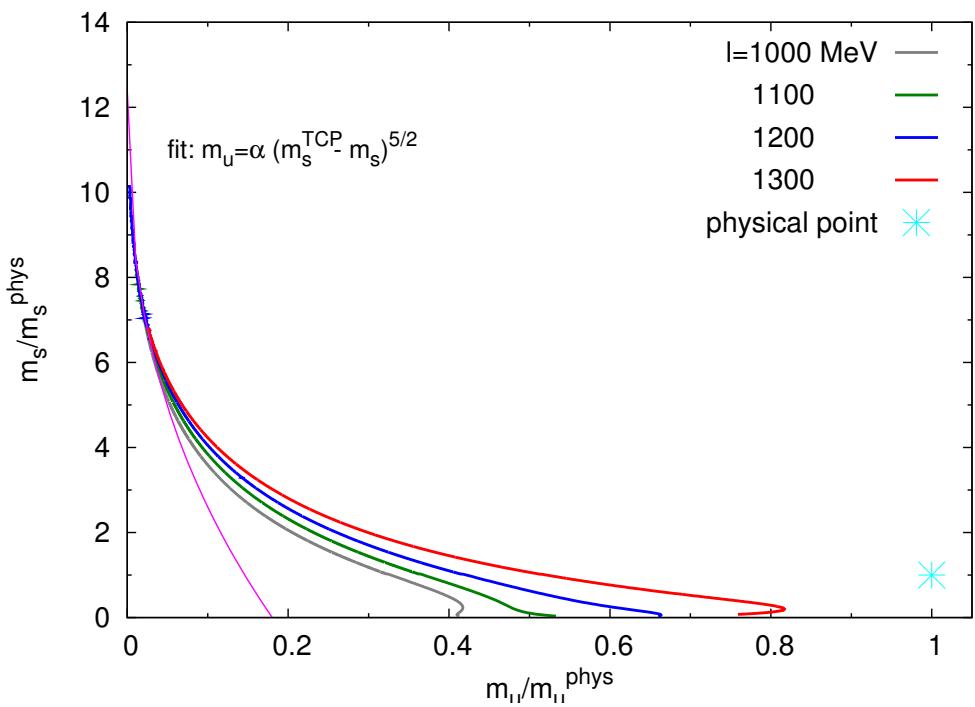
the location of the TCP: $m_K^{TCP} \in (1700, 1850) \text{ MeV}$



Veneziano formula: $m_\eta^2 = m_K^2 + \frac{1}{2} \Delta m_{\eta 0}^2 - \frac{1}{2} [(2m_K^2 - 2m_\pi^2 - \frac{1}{3} \Delta m_{\eta 0}^2)^2 + \frac{8}{9} \Delta m_{\eta 0}^4]^{1/2}$
 $\Delta m_{\eta 0}^2$ non-perturbative gluonic contribution

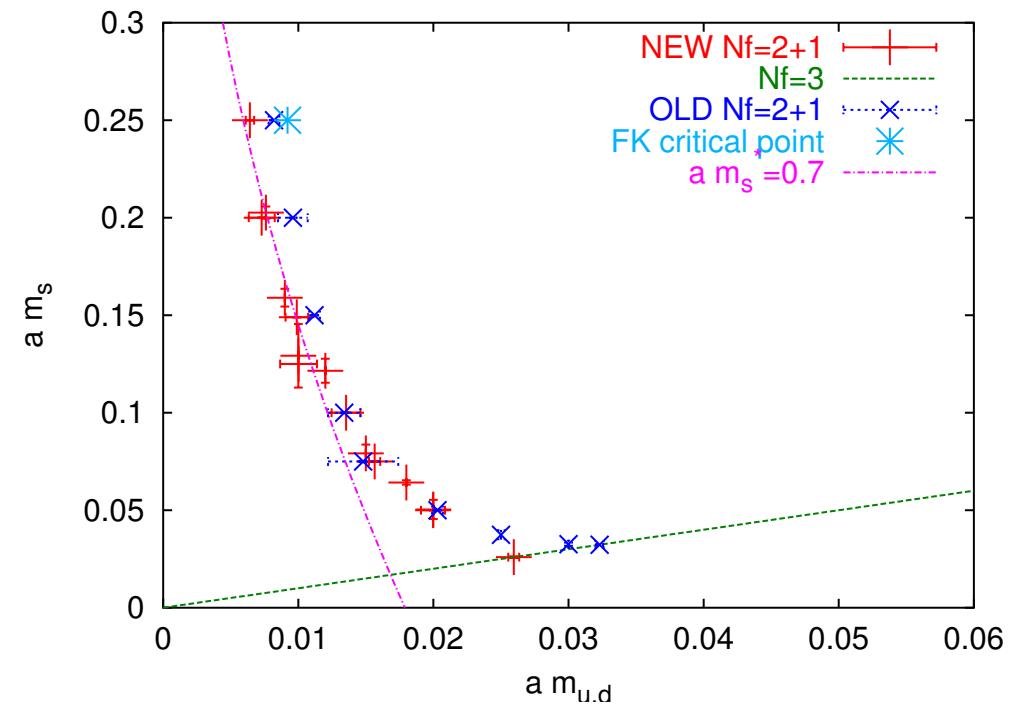
Comparison with lattice

T. Herpay, Zs. Sz., hep-ph/0604086



$$m_s/m_s^{\text{Phys}} \simeq 13$$

O. Philipsen, P. de Forcrand, hep-lat/0409034



$$m_s/m_s^{\text{Phys}} \simeq 3$$

message: the scaling region of TCP $m_{u,d} \approx (m_s^{\text{TCP}} - m_s)^{5/2}$ sets in far away from the physical point, close to the $m_{u,d} = 0$ axis

Conclusions

- improved determination of the phase-boundary achieved by taking into account the $m_\pi - m_K$ -dependence of the couplings with the help of ChPT
- the estimate for the location of the boundary for $m_\pi = m_K$ is compatible with the accuracy of current lattice results

quasi-particle approximation: $m_\pi^c = 40 \pm 20 \text{ MeV}$

complete renormalized one-loop: $m_\pi^c = 110 \pm 20 \text{ MeV}$

- the location of the tricritical point on the $m_{u,d} - m_s$ -plane is renormalization scale independently estimated at $m_s^{\text{TCP}} = (13 - 15) \times m_s^{\text{phys}}$
- self-consistent approximation (2PI, Dyson-Schwinger) required to test the validity of the result